Full AC network Integrated Core Solver for the SuperOPF Framework

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Issues with Current Generation of Optimal Power Flow

- Optimal power flow solution is NOT a global optimal solution
- Solvers only compute one (local) optimal solution while there are multiple local optimal solutions
- Each OPF solution corresponds to one location marginal pricing (which OPF solution is the right one ?)
- Current solvers are still not sufficiently robust
- Current solvers still can not correctly handle stability constraints

Challenges: Problem Formulations and Solvers

$$\min C(x)$$

Subject to: $h(x) = 0$

$g(x) \le 0$

However, security-constrained OPF can not be expressed as the above analytical form:

i. Power balance equations: $h(x) = 0$

ii. Voltage limit constraints: $\underline{x} \le x \le \overline{x}$

iii. Thermal limit constraints: $g(x) \le 0$

iv. Transient-stability constraints: ???

v. Voltage stability constraints: ???

Implications:
(i) It is not possible to represent them in explcit forms.
(ii) approximations are subject to incorrect stability assessment.

i. *Transient-stability constraints:* $h(x) = 0$

ii. *Voltage stability constraints:* $g(x) \leq 0$

Super-OPF (for operation)

Super-OPF Method

Input Data → **Stage I: OPF Constraint Analyzer** → **Stage II: Simple OPF w/o Thermal Limits** → **Stage III: Homotopy OPF w/ Active Thermal Limits** → **Stage IV: Sensitivity Analyzer for Discretization** → OPF Result

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0,\ i=1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0,\ i=1,\cdots,n_B \\
& \underline{V}_i \le V_i \le \overline{V}_i,\ i=1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \overline{t}_i,\ i=1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \overline{\phi}_i,\ i=1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \overline{b}_i,\ i=1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \overline{P}_j^G,\ j=1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \overline{Q}_j^G,\ j=1,\cdots,n_G
\end{aligned}
$$

OPF without thermal constraints

$$
\begin{aligned}
\min \quad & f(V,\theta,t,\phi,b,P^G,Q^G;\lambda) \\
s.t. \quad & P_i(V,\theta,t,\phi,b) + P_i^L - P_i^G = 0,\ i=1,\cdots,n_B \\
& Q_i(V,\theta,t,\phi,b) + Q_i^L - Q_i^G = 0,\ i=1,\cdots,n_B \\
& S_{ij}(V,\theta,t,\phi,b) \le \overline{S}_{ij} + \lambda\tilde{S}_{ij},\ (i,j)\in\mathcal{L}_\alpha \\
& S_{ji}(V,\theta,t,\phi,b) \le \overline{S}_{ij} + \lambda\tilde{S}_{ij},\ (i,j)\in\mathcal{L}_\alpha \\
& \underline{V}_i \le V_i \le \overline{V}_i,\ i=1,\cdots,n_B \\
& \underline{t}_i \le t_i \le \overline{t}_i,\ i=1,\cdots,n_T \\
& \underline{\phi}_i \le \phi_i \le \overline{\phi}_i,\ i=1,\cdots,n_P \\
& \underline{b}_i \le b_i \le \overline{b}_i,\ i=1,\cdots,n_S \\
& \underline{P}_j^G \le P_j^G \le \overline{P}_j^G,\ j=1,\cdots,n_G \\
& \underline{Q}_j^G \le Q_j^G \le \overline{Q}_j^G,\ j=1,\cdots,n_G
\end{aligned}
$$

OPF with active thermal constraints

$$
\begin{aligned}
S_y^f &= \frac{\partial f}{\partial y} - \left(\frac{\partial g}{\partial y}\right)^T \left[\left(\frac{\partial g}{\partial x}\right)^T\right]^{-1} \frac{\partial f}{\partial x} \\
S_y^h &= \frac{\partial h}{\partial y} - \frac{\partial h}{\partial x}\left(\frac{\partial g}{\partial x}\right)^{-1}\frac{\partial g}{\partial y}
\end{aligned}
$$

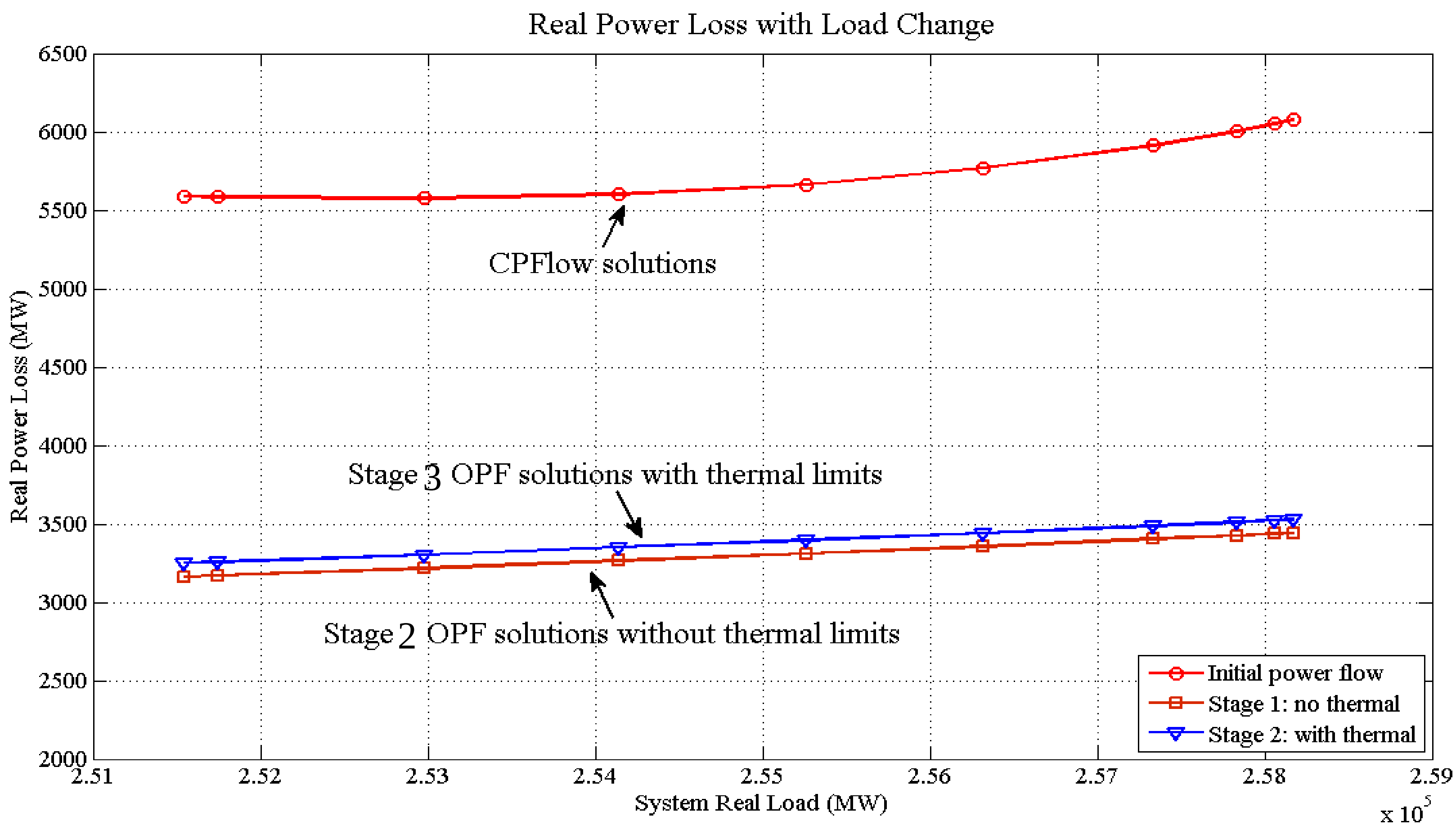
$$
\begin{aligned}
\Delta f_i^+ &= S_{d_i}^f (d_i^{j_i+1} - d_i^{j_i}) \\
\Delta f_i^- &= S_{d_i}^f (d_i^{j_i-1} - d_i^{j_i}) \\
\Delta h_{k_i}^+ &= S_{d_i}^{h_k} (d_i^{j_i+1} - d_i^{j_i}),\ \forall k = 1,\cdots,n_h \\
\Delta f_{k_i}^- &= S_{d_i}^{h_k} (d_i^{j_i+1} - d_i^{j_i}),\ \forall k = 1,\cdots,n_h
\end{aligned}
$$

$$
\begin{aligned}
\eta_i^+ &= w_f \Delta f_i^+ + \textstyle\sum_{k=1}^{n_h} w_h \max[0, h_k(\hat{x},\hat{y}) + \Delta h_{k_i}^+] \\
\eta_i^- &= w_f \Delta f_i^- + \textstyle\sum_{k=1}^{n_h} w_h \max[0, h_k(\hat{x},\hat{y}) + \Delta h_{k_i}^-]
\end{aligned}
$$

$$
\begin{aligned}
& y_i^{j_i} \leftarrow y_i^{j_i+1} \text{ if: } \eta_i^+ \le \eta_i^- \text{ and } \eta_i^+ \le \eta_{th} \text{ or} \\
& y_i^{j_i} \leftarrow y_i^{j_i-1} \text{ if: } \eta_i^- < \eta_i^+ \text{ and } \eta_i^- \le \eta_{th}.
\end{aligned}
$$

Sensitivity based adjustment

Results: Real Power Loss



Super-OPF Dimensions

13183-Bus System

Input Data → Stage I: OPF Constraint Analyzer → Stage II: Simple OPF w/o Thermal Limits → Stage III: Homotopy OPF w/ Active Thermal Limits → Stage IV: Sensitivity Analyzer for Discretization → OPF Result

System:
- Buses: 13183
- Loads: 9691
- Generators: 2304
- Branches: 18168
- Transformers: 1410
- Switched shunts: 1404

OPF Dimensions:
- Dimension of x: 31134
- Nonlinear equality constraints: 26366
- Nonlinear inequality constraints: 0
- Total equality constraints: 26367
- Total inequality constraints: 35902

OPF Dimensions:
- Dimension of x: 31134
- Nonlinear equality constraints: 26366
- Nonlinear inequality constraints: varying (<100)
- Total equality constraints: 26367
- Total inequality constraints: >35902 (varying)

System:
- Continuous variables: 28320
- Discrete variables: 2814

Results: Efficiency and Robustness (Analytical Jacobian matrices)

Effects of constraint analysis

Base case
Without constraint analysis
• Converged in 217 iterations • CPU time: 177 seconds • OPF loss: 3251.284MW
With constraint analysis
• Converged in 191 iterations • CPU time: 143 seconds • OPF loss: 3251.353MW

Robustness of our method

Loading Condition	One-Staged Scheme	Multi-Staged Scheme
1	Succeeded	Succeeded
2	Succeeded	Succeeded
3	Succeeded	Succeeded
4	Succeeded	Succeeded
5	Failed	Succeeded
6	Failed	Succeeded
7	Failed	Succeeded
8	Failed	Succeeded
9	Failed	Succeeded
10	Failed	Succeeded

Adaptive Homotopy-guided, Active-Set-Assisted Primal-Dual Interior Point OPF Solver

Homotopy-based Methodology (continuation method + adaptive step-size)

Domain-knowledge-based

Active-set assisted

IPM methods consists of three basic modules:

- Newton method solving nonlinear equations
- Lagrange’s method for optimization with equality constraints.
- barrier method for optimization with inequalities.

A Robust AC OPF Solver

1. (support industrial model) A commercial-grade core SuperOPF software supporting various industrial-grade power system models such as
(i) CIM-compliance; and
(ii) PSS/E data format

2. A multi-stage OPF solver with adaptive homotopy-based Interior Point Method for large-scale power systems (14,000-bus data)

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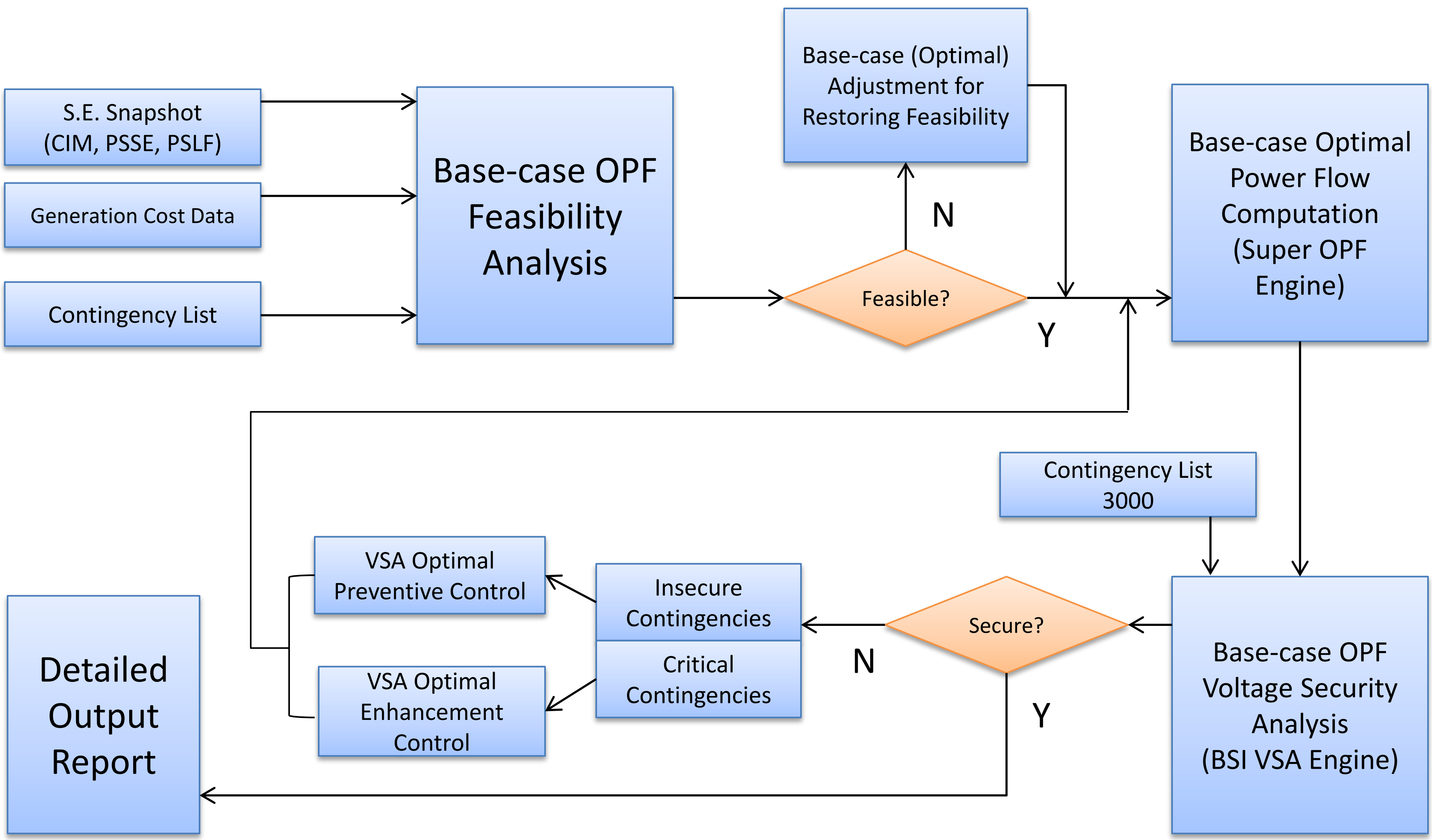
Super-OPF-VS (Voltage Stability) (Phase II)

1. Input 2. Feasibility Check 3. Ensuring Feasibility 4. Computation Engine



7. Output Report 6. VSA Enhancement 5. VSA Check

Super-OPF Contingency Analysis

A practical 6534-Bus System

Initial Power Flow from State Estimator → Super-OPF Solution → BSI VSA Preventive Control

1062 N-1 Contingencies

0	0	3010.028	-20.088	0	3010.028
1	21	-0.305	-0.305	3	0.000
2	34	-0.218	-0.218	6	0.000
3	691	-0.100	-0.100	17	0.000
4	8	-0.066	-0.066	1	0.000
5	35	-0.060	-0.060	5	0.000
6	24	-0.056	-0.056	2	0.000
7	36	-0.015	-0.015	4	0.000
8	639	1353.153	-9.031	12	2347.311
9	281	1884.894	-12.579	10	1397.252
10	282	1884.894	-12.579	11	1397.252
11	491	2715.639	-18.123	14	2481.276
12	492	2715.639	-18.123	13	2481.276
13	561	2716.463	-18.129	15	2485.825
14	521	2730.695	-18.224	16	2503.604
15	572	2789.834	-18.619	41	2789.834
16	864	2791.814	-18.632	42	2791.814
17	863	2791.814	-18.632	43	2791.814
18	628	2805.778	-18.725	44	2805.778
19	690	2807.721	-18.738	18	2619.026
20	684	2822.604	-18.837	47	2822.604

Load margin: 3010MW
Objective (loss): 2793.6MW
7 insecure contingencies

0	0	4840.000	-32.301	0	4840.000
1	21	-0.285	-0.285	3	0.000
2	34	-0.253	-0.253	5	0.000
3	35	-0.071	-0.071	4	0.000
4	8	-0.050	-0.050	1	0.000
5	24	-0.042	-0.042	2	0.000
6	561	4373.778	-29.189	16	4304.356
7	491	4386.839	-29.276	15	4284.000
8	492	4386.839	-29.276	14	4284.000
9	691	4404.315	-29.393	20	4404.315
10	521	4407.446	-29.414	17	4316.588
11	690	4437.187	-29.612	23	4437.187
12	281	4494.606	-29.996	18	4381.087
13	282	4494.606	-29.996	19	4381.087
14	639	4508.597	-30.089	21	4413.366
15	428	4544.092	-30.326	25	4483.436
16	432	4544.092	-30.326	26	4483.436
17	431	4544.092	-30.326	27	4483.436
18	430	4544.092	-30.326	28	4483.436
19	429	4544.092	-30.326	29	4483.436
20	501	4555.107	-30.399	30	4511.405

Load margin: 4840MW
Objective (loss): 1642.8MW
5 insecure contingencies

0	0	4840.161	-32.302	0	4840.161
1	561	4376.021	-29.204	23	4303.308
2	492	4389.094	-29.291	22	4282.792
3	491	4389.094	-29.291	21	4282.792
4	521	4409.681	-29.429	24	4315.462
5	690	4436.558	-29.608	25	4436.558
6	691	4509.283	-30.094	6	2244.418
7	431	4546.083	-30.339	26	4482.817
8	430	4546.083	-30.339	27	4482.817
9	429	4546.083	-30.339	28	4482.817
10	428	4546.083	-30.339	29	4482.817
11	432	4546.083	-30.339	30	4482.817
12	501	4556.994	-30.412	31	4510.822
13	509	4564.158	-30.460	32	4514.185
14	281	4571.971	-30.512	14	2416.908
15	282	4571.971	-30.512	15	2416.908
16	506	4577.873	-30.551	33	4534.745
17	662	4580.186	-30.567	38	4550.269
18	663	4580.186	-30.567	39	4550.269
19	456	4585.097	-30.599	34	4536.546
20	455	4585.097	-30.599	35	4536.546

Load margin: 4840MW
Objective (loss): 1674.6MW
No insecure contingency

This phase is focused on the following enhancements

Topicss	Enhancements
Co-optimization over multiple scenario(functions)	(i) deal with multiple base-cases (i.e., co-optimize multiple base-cases)
Commercial-grade packages (applications)	(ii) deal with thermal limits and voltage limits under AC power flow models of a large set of contingencies.
Renewables (uncertainties)	(iii) deal with uncertainties of wind generations and other renewables

This phase is focused on the following enhancements

Topics	Enhancements
Co-optimization over multiple scenario(functions)	(vii) Engage utility companies to provide their assessment of and interest in adopting SuperOPF.
Commercial-grade packages (applications)	(viii) Engage utility companies to assist the development of SuperOPF.
Uncertainties in contingencies	(viiii) Co-optimized SuperOPF-Static + renewables + contingency package

SuperOPF Co-optimization

- Objective: minimizing the expected cost across all the scenarios

$$
\begin{aligned}
min \quad & f(x) = f_0(x_0) + \sum_{k=1}^{K} p_k[f_k(x_k) + c_k(x_k - x_0)] \\
s.t. \quad & h_0(x) = 0 \\
& g_0(x) \leq 0 \\
& \cdots \cdots \\
& h_K(x) = 0 \\
& g_K(x) \leq 0
\end{aligned}
$$

$x = (x_0, x_1, \cdots, x_K)$: optimization variables $\quad$ p_k: probability for k-th scenario

$x_i = (\Theta^k, V^k, T^k, S^k, B^k, P_G^k, Q_G^k)$: variables of the k-th scenario (0: base case)

$f_0(x_0)$: base case cost $\quad$ $f_k(x_k)$: k-th base cost (reserves, load shedding, etc)

$c_k(x_k - x_0)$: cost of scenario-induced deviations (from base-case)

SuperOPF Co-optimization

Four types of scenarios

Type-1 scenario: Base case

$$
\begin{aligned}
\min \quad & f(x) & \\
s.t. \quad & P_i(x) + P_{Di} - P_{Gi} = 0 & 1 \le i \le n_B \\
& Q_i(x) + Q_{Di} - Q_{Gi} = 0 & \\
& S_k = \sqrt{P_{ij}^2(x) + Q_{ij}^2(x)} \le S_k^{max} & (i,j) \in L \\
& x^{min} \le x \le x^{max} &
\end{aligned}
$$

n_B: the number of buses L: the set of branches

Type-2 scenario: Base case + contingency

$$
\begin{aligned}
\min \quad & f(x) & \\
s.t. \quad & P_i(x) + P_{Di} - P_{Gi} = 0 & 1 \le i \le n_B \\
& Q_i(x) + Q_{Di} - Q_{Gi} = 0 & \\
& S_k = \sqrt{P_{ij}^2(x) + Q_{ij}^2(x)} \le S_k^{max} & (i,j) \in \hat{L} \\
& x^{min} \le x \le x^{max} &
\end{aligned}
$$

$\hat{L}$: L excludes contingent branches

Type 3 scenario: Base case + renewable energy

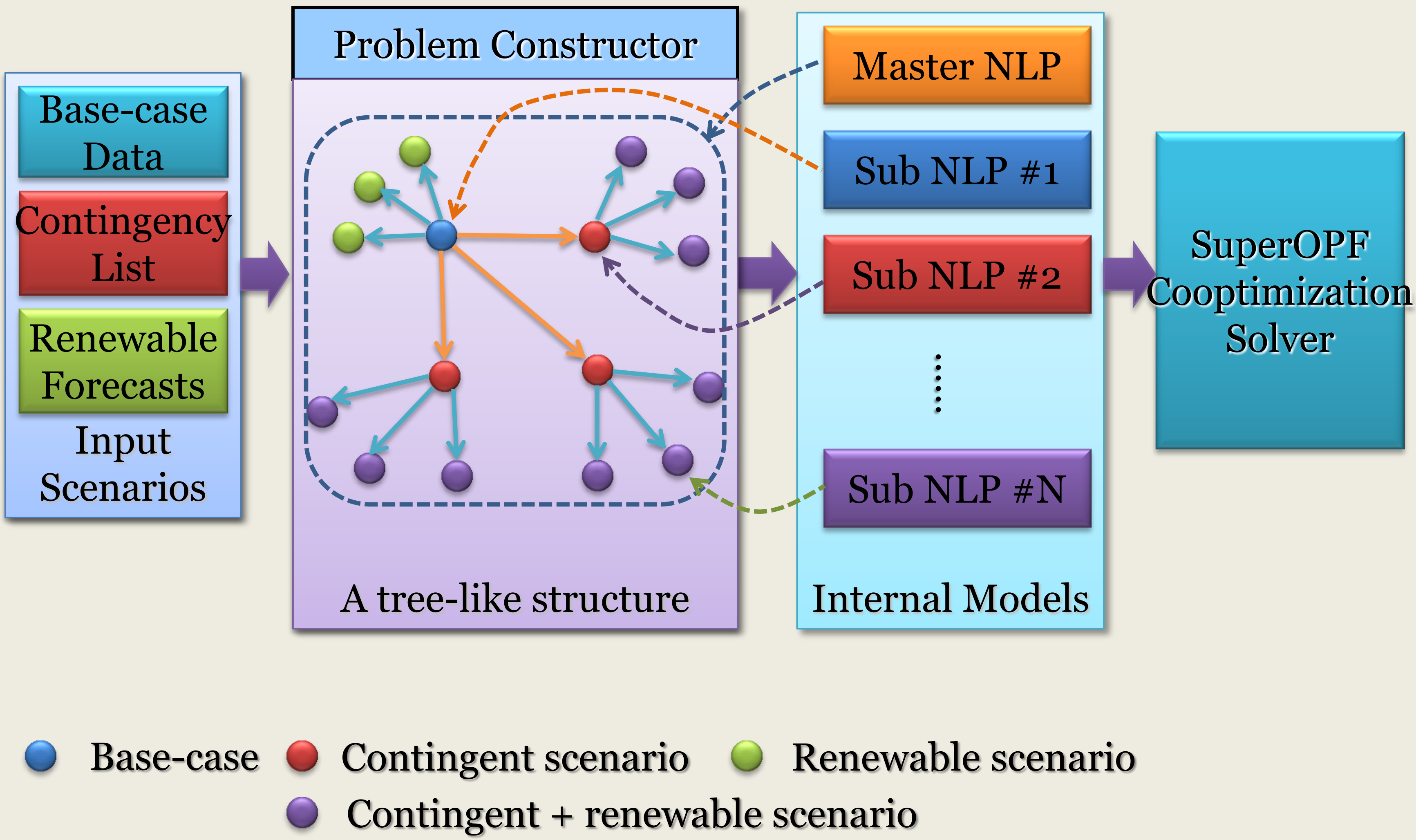
$$
\begin{aligned}
\min \quad & f(x) & \\
s.t. \quad & P_i(x) + \hat{P}_{Di} - P_{Gi} = 0 & 1 \le i \le n_B \\
& Q_i(x) + \hat{Q}_{Di} - Q_{Gi} = 0 & \\
& S_k = \sqrt{P_{ij}^2(x) + Q_{ij}^2(x)} \le S_k^{max} & (i,j) \in L \\
& x^{min} \le x \le x^{max} &
\end{aligned}
$$

Type 4 scenario: Base case + renewable energy + contingency

$$
\begin{aligned}
\min \quad & f(x) & \\
s.t. \quad & P_i(x) + \hat{P}_{Di} - P_{Gi} = 0 & 1 \le i \le n_B \\
& Q_i(x) + \hat{Q}_{Di} - Q_{Gi} = 0 & \\
& S_k = \sqrt{P_{ij}^2(x) + Q_{ij}^2(x)} \le S_k^{max} & (i,j) \in \hat{L} \\
& x^{min} \le x \le x^{max} &
\end{aligned}
$$

$\hat{P}_D, \hat{Q}_D$: equivalent loads with renewable energies

SuperOPF Co-optimization

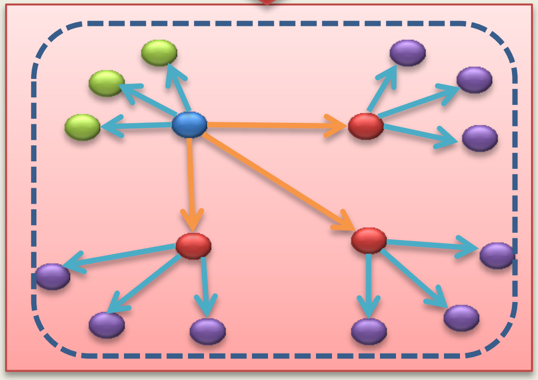


SuperOPF Co-optimization

Base Case → Contingencies + Renewable Energy Forecasts → Internal Combinatorial Scenarios $K=(M+1)*(N+1)$ → SuperOPF Co-optimization

$$
\begin{array}{ll}
min & f_0(P_0) \\
s.t & h_0(x) = 0 \\
 & g_0(x) \leq 0
\end{array}
$$

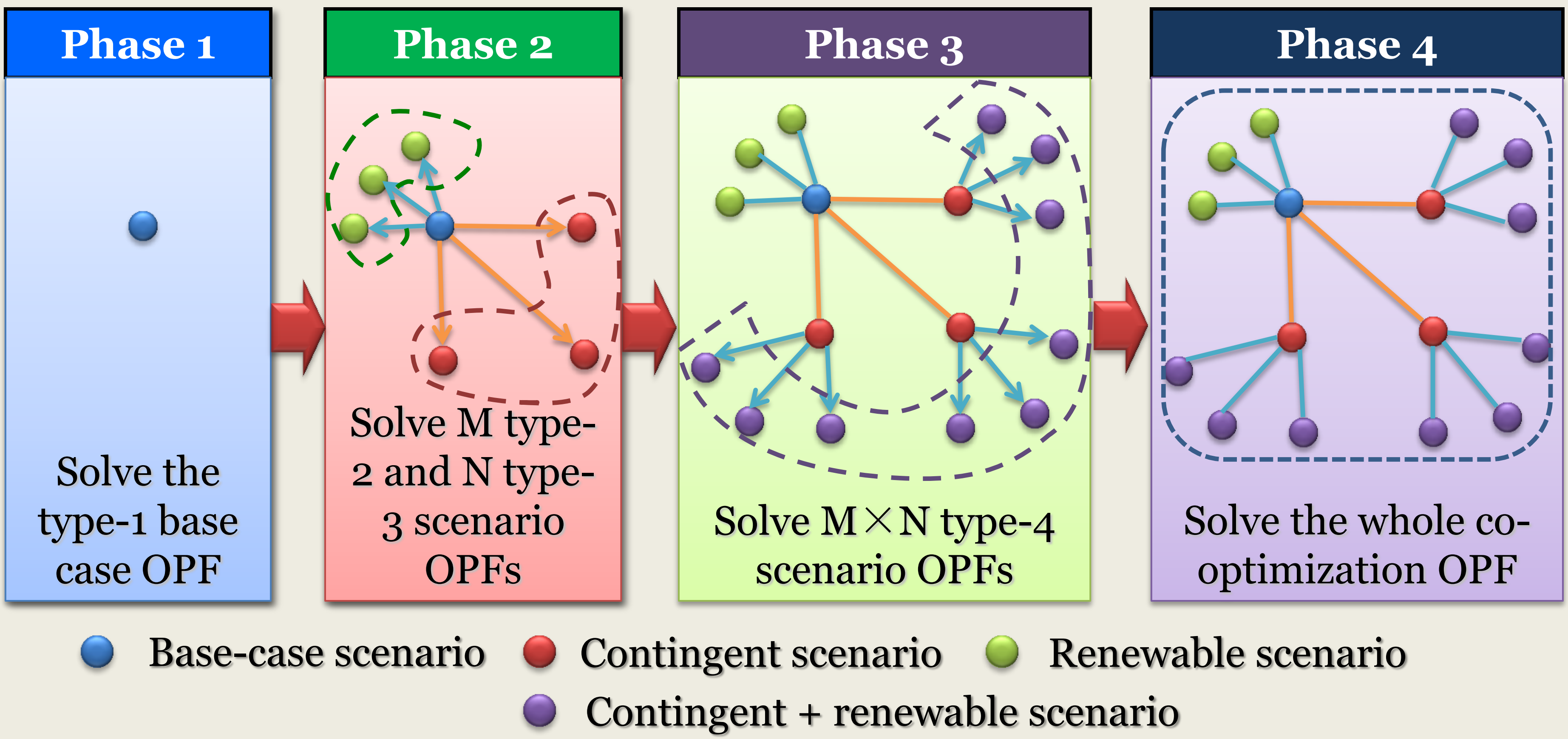
M: # of contingencies
N: # of renewable energy forecasts



$$
\begin{array}{ll}
min & f(x) = f_0(x_0) + \sum_{k=1}^{K} p_k[f_k(x_k) + c_k(x_k - x_0)] \\
s.t. & h_0(x) = 0 \\
 & g_0(x) \leq 0 \\
 & \dots\dots \\
 & h_K(x) = 0 \\
 & g_K(x) \leq 0
\end{array}
$$

Multi-phase Approach

- A multi-phase scheme is developed in which base case OPF solutions are used as initial points for solving scenario problems. A combination of all sub-problem solutions is used as the initial point for the entire co-optimization problem.



Optimization Variables

- All or a subset of:
  - Voltage magnitudes and phase angles
  - Real and reactive power generations
  - Transformer tap ratios (continuous or discrete)
  - Phase shifters (continuous or discrete)
  - Switchable shunts (continuous or discrete)
  - Load shedding

Supported Scenario Types

- Contingent scenarios
  - Disconnection of branches
  - Removal of generators
  - Removal of shunts
  - Removal of loads
- Renewable forecast scenarios
- Combination of contingent and renewable forecast scenarios

Numerical Simulations

- Two practical large-scale systems
  - a 6534-bus system
  - A 13183-bus system

- Simulation environment:
  - 2.6GHz quad-core Intel i7-3720QM processor (Turbo boost to 3.6GHz), 16GB 1600MHz DDR3 RAM, Mac OSX 10.8.4, GCC 4.8.1

Co-optimization Results on a 6500-bus System

Base Case → Contingencies + Renewable Energy Forecasts → Internal Combinatorial Scenarios → SuperOPF Co-optimization

System:

Buses: 6534

Loads: 2901

Generators: 1903

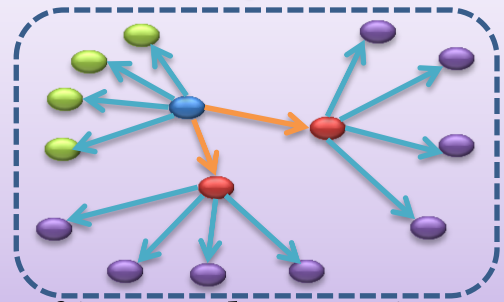
Branches: 8295

Transformers: 294

Switchable shunts: 520

# of contingencies: 2

# of renewable forecasts: 4



# of internal scenarios: 15

- # of optimization variables: 226,215;
- # of constraints: 287,360 (equality: 181,260; inequality: 106,100);
- # of Hessian non-zeros: 2,199,566

Numerical Simulations

- Simulated scenarios
  - Contingencies (N-1):
    - Removal of a single randomly selected branch from the network (ensuring without resulting islands or isolated buses)
  - Renewable energy forecasts:
    - Wind generators: random selection of 20% system generators;
    - Forecasts: random outputs varying uniformly in the range of ±25% of the initial outputs. Each set of forecasts assigned a probability in 1%~10%.

Co-optimization Results on CAISO System

Sub-problem	Scenario	*p*	F(x)	# of Iters	CPU Time (sec)	Sub-problem	Scenario	*p*	F(x)	# of Iters	CPU Time (sec)
	Initial PF		80.418985			9	Base case + Renewable 2 + Contingency 2	0.573%	21.076384	79	10.38
1	Base case		21.085284	80	10.48	10	Base case + Renewable 3	1.28%	21.089217	77	10.11
2	Base case + Contingency 1	10%	21.106819	79	10.52	11	Base case + Renewable 3 + Contingency 1	0.128%	21.110689	74	9.70
3	Base case + Contingency 2	10%	21.085464	78	10.32	12	Base case + Renewable 3 + Contingency 2	0.128%	21.089421	77	10.09
4	Base case + Renewable 1	3.04%	21.172620	79	10.46	13	Base case + Renewable 4	5.60%	21.218402	75	9.88
5	Base case + Renewable 1 + Contingency 1	0.304 %	21.194469	80	10.43	14	Base case + Renewable 4 + Contingency 1	0.560%	21.240219	78	10.31
6	Base case + Renewable 1 + Contingency 2	0.304 %	21.173087	80	10.58	15	Base case + Renewable 4 + Contingency 2	0.560%	21.218608	76	10.26
7	Base case + Renewable 2	5.73%	21. 076129	83	10.92	Cooptimization problem			21.129602	310	2281.02 (i.e. 38 min.
8	Base case + Renewable 2 + Contingency 1	0.573%	21. 097419	82	10.85						

Co-optimization Results on a practical System

Base Case → Contingencies + Renewable Energy Forecasts → Internal Combinatorial Scenarios → SuperOPF Co-optimization

System Description:

Buses: 13183

Loads: 9691

Generators: 2304

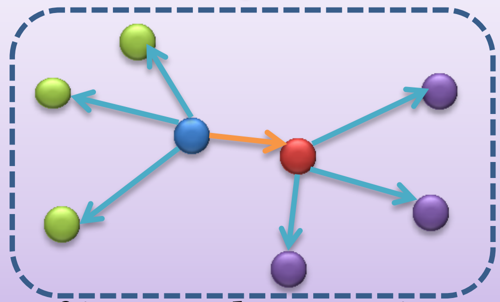
Branches: 18168

Transformers: 1410

Switched shunts: 1404

# of contingencies: 1

# of renewable forecasts: 3



# of internal scenarios: 8

- # of optimization variables: 249,072;
- # of constraints: 309,016 (equality: 210,928; inequality: 98,088);
- # of Hessian non-zeros: 3,826,508.

Co-optimization Results

Sub-problem	Scenario	*p*	F(x)	# of Iters	CPU Time (sec)	Sub-problem	Scenario	*p*	F(x)	# of Iters	CPU Time (sec)
	Initial PF		167.06924			5	Base case + Renewable 2	9.92%	67.875872	196	71.14
1	Base Case		67.959196	177	64.26	6	Base case + Renewable 2 + Contingency 1	0.992%	67.875573	290	106.07
2	Base case + Contingency 1	10%	67.958466	214	77.74	7	Base case + Renewable 3	9.01%	67.905719	216	78.71
3	Base case + Renewable 1	3.24%	68.053362	224	83.08	8	Base case + Renewable 3 + Contingency 1	0.901%	67.905028	183	66.54
4	Base case + Renewable 1 + Contingency 1	0.324%	68.052682	340	123.31	Cooptimization problem (using the 4-phase scheme)			67.972861	478	5582.25 (or 93 min.

1-shot scheme: cannot converge after 1000 iterations (about 5 hours)!

Complexity Analysis

- Rough calculation

15 X 15 = 225, 10 sec. X 225 = 2250 sec.

- Computation complexity increases quadratically with the number of scenarios. Hence, the task of scenario reduction is important.

Observations

- SuperOPF solver can successfully solve multi-scenario co-optimization problems on large scale power systems.
- Complexity of the co-optimization problem grows considerably as the number of scenarios increases.
- Scenario reduction schemes are needed for SuperOPF in solving large-size problems.

Proposed Requirements for Scenario Reduction Schemes

(reliability measure) identify all representative scenarios that properly maintain important information of stochastic variables.

(efficiency measure) the retain important information with the least number of scenarios.

(*speed and robust measure*) It should be fast and robust to operating conditions

Scenario Reduction Techniques

- Forward selection and backward reduction are the most used scenario reduction technique.
- These methods all focus on :

"distance" between the selected scenario set and the original scenario set. They are problem-independent.

Our Proposed Scenario Reduction Scheme for Voltage Stability

- Problem-dependent

Application-Oriented Scheme

Voltage Stability Analysis Under a large number of scenarios

Stage I: Cluster Scheme (input space)

Stage II: Screening and Ranking Scheme (effect, output space)

Stage III: Detailed Analysis (effect, output space)

Voltage Stability Analysis Under Uncertainty (Cluster + Screening + ranking + detailed analysis)

In comparison with Monte Carlo method (Scenario : 5000)

IEEE 118-bus Test System	
(Renewables at 1, 7, 40, 78, 117)	Weibull distribution

Reduction Ratio	Accuracy(%)	Missing Scenarios
99.08%	100%	0

Scenarios	5000
Stage I & II	Reduce to 46 scenarios
Stage III	Reduce to 17 scenarios

Voltage Stability Analysis Under Uncertainty (Cluster + Screening + ranking + detailed analysis)

Poland 3120-bus	
(23, 68, 69, 70, 261, 263, 1393, 1395, 1398, 3100, 3101, and 3102)	Weibull distribution

Reduction Ratio	Accuracy(%)	Critical Missing Scenarios
98.52%	100%	0

Scenarios	5000
Stage I & II	Reduce to 74 scenarios
Stage III	Reduce to 29 scenarios

Scenario Reduction Scheme for OPF

- Problem-dependent

Application-Oriented Scheme

Voltage Stability Analysis Under a large number of scenarios

Stage I: Cluster Scheme (input space)

Stage II: Screening and Ranking Scheme (effect, output space)

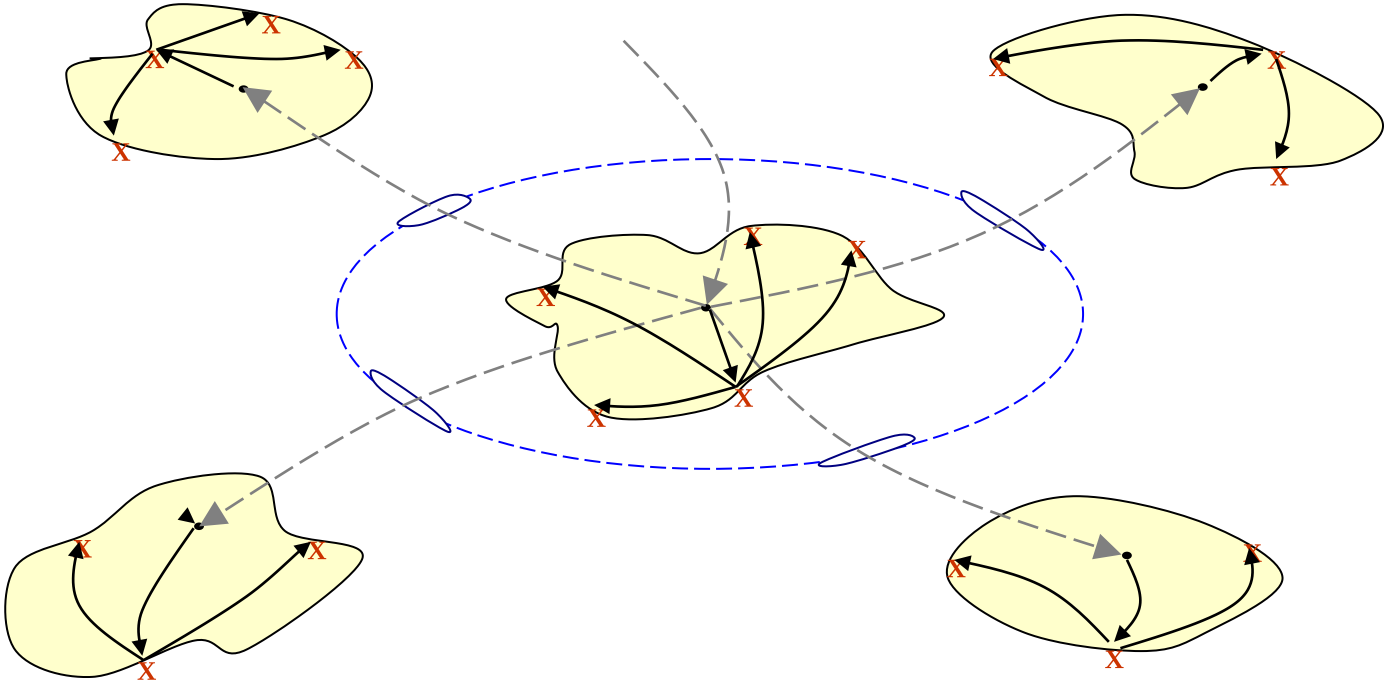
Stage III: Detailed Analysis (effect, output space)

Issues with Current Generation of Optimal Power Flow

- Optimal power flow solution is NOT a global optimal solution
- Solvers only compute one (local) optimal solution while there are multiple local optimal solutions
- Each OPF solution corresponds to one location marginal pricing (which OPF solution is the right one ?)
- Current solvers are still not sufficiently robust
- Current solvers still can not correctly handle stability constraints

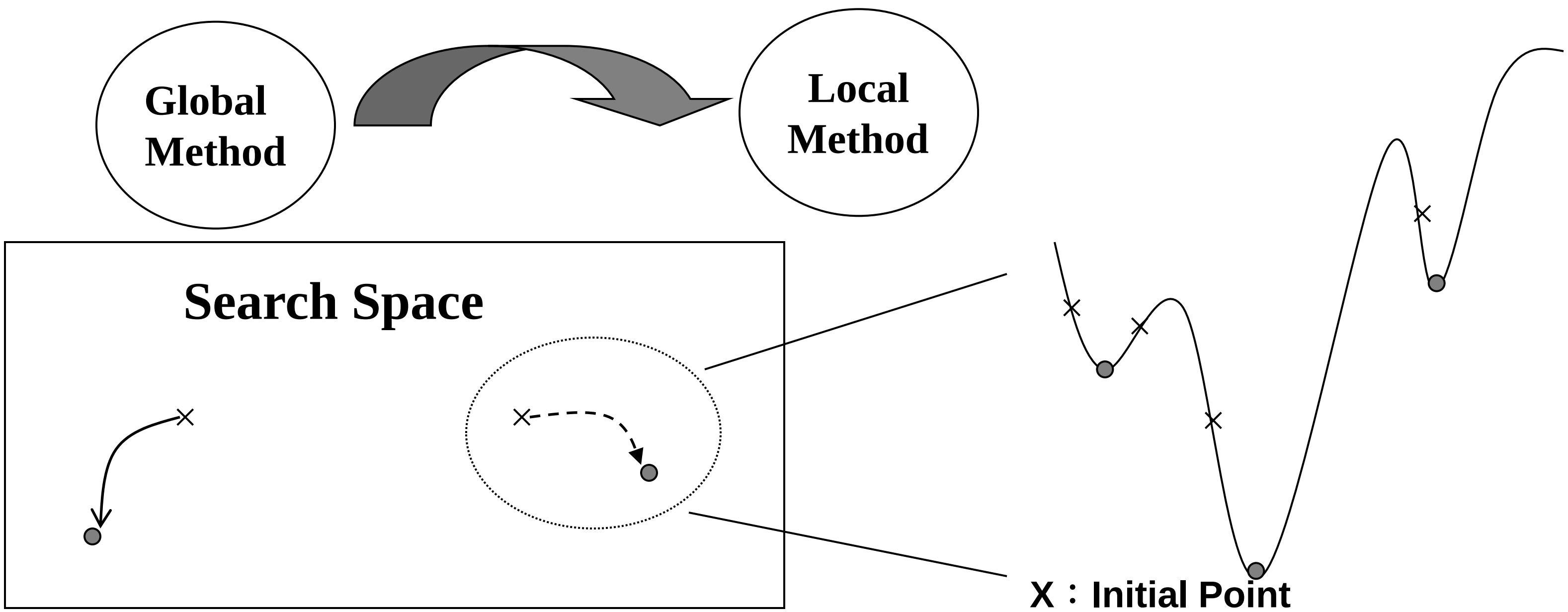
Multiple Optimal Solutions

(1) There are multiple feasible components
(2) Multiple local optimal solutions in each feasible component



TRUST-TECH

- TRUST-TECH—A Commander for the existing optimization methods：



X：Initial Point

●：Local Optimal Solution

X：Initial Point

●：Local Optimal Solution

Issue：

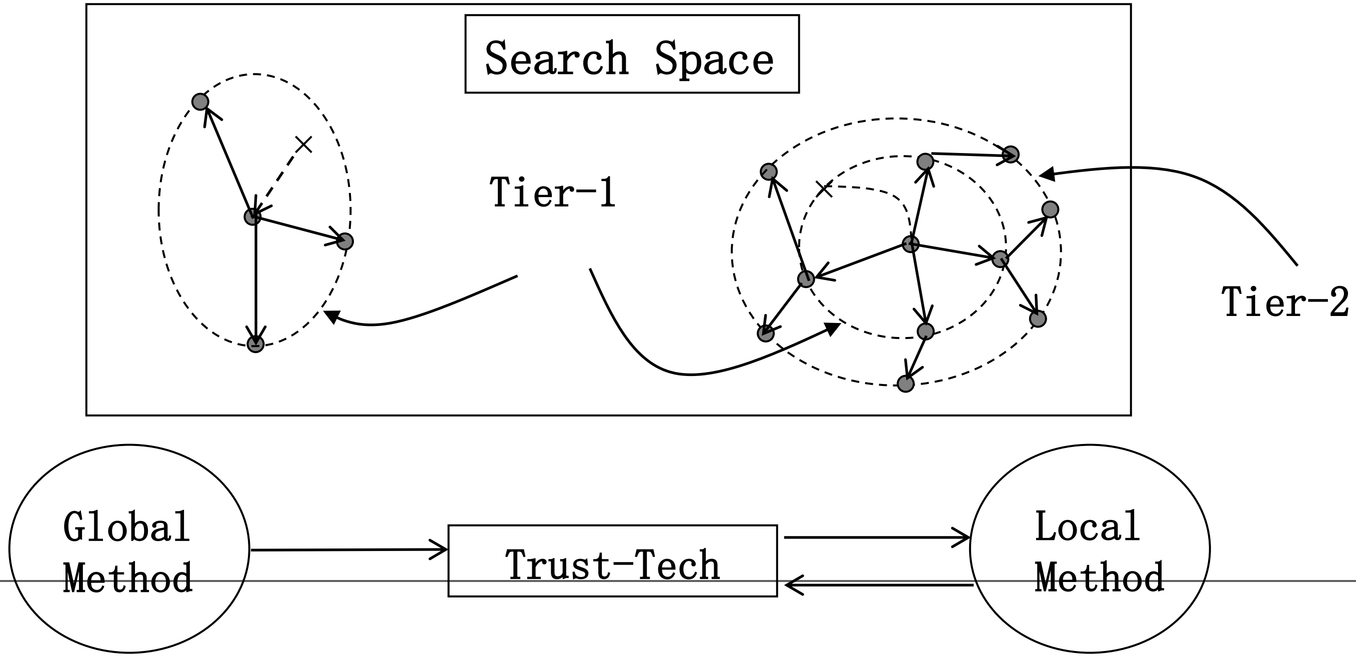
(i) Initial Condition

(ii) Convergence Region of Local Method

TRUST-TECH

TRUST-TECH Methodology

- It has a systematic and deterministic process to find multiple local optimal solutions; i.e. in a tier-by-tier manner with tier-1 local optimal solutions and then higher-tier local optimal solutions, etc.



Development of TRUST-TECH I

	Train XP = Trust-Tech			Test XP = Trust-Tech		
	Best BP	BP+XP	Improvement(%)	Best BP	BP+XP	Improvement(%)
Cancer	2.21	1.74	27.01	3.95	2.63	50.19
Image	9.37	8.04	16.54	11.08	9.74	13.76
Ionosphere	2.35	0.57	312.28	10.25	7.96	28.77
Iris	1.25	1.00	25.00	3.33	2.67	24.72
Diabetes	22.04	20.69	6.52	23.83	20.58	15.79
Sonar	1.56	0.72	116.67	19.17	12.98	47.69
Wine	4.56	3.58	27.37	14.94	6.73	121.99

Development of TRUST-TECH I

	Train XP = Trust-Tech			Test XP = Trust-Tech		
	Best GA	GA+XP	Improvement(%)	Best GA	GA+XP	Improvement(%)
Cancer	2.69	1.87	43.85	3.79	2.77	36.82
Image	13.08	10.09	29.63	14.72	12.81	14.91
Ionosphere	3.27	1.07	205.61	10.83	8.26	31.11
Iris	1.58	1.25	26.40	2.67	2.67	0.00
Diabetes	31.95	28.55	11.91	33.59	31.24	7.52
Sonar	9.55	0.36	2552.78	23.6	16.31	44.70
Wine	12.68	3.44	268.60	16.99	6.18	174.92

Introduction of TRUST-TECH

- Paper-Theory and Application
  - Bin Wang, **Hsiao-Dong Chiang**. ELITE: Ensemble of Optimal, Input-Pruned Neural Networks Using TRUST-TECH. *IEEE Transactions on Neural Networks*, 22(1): 96-109, 2011.
  - **Hsiao-Dong Chiang**, Bin Wang, Quan-Yuan Jiang. Applications of Trust-Tech Methodology in Optimal Power Flow of Power Systems. *Optimal Operations of Energy Systems*, Springer, International series in operations research and Management Science, 2009.
  - **Hsiao-Dong Chiang**, J-H Chen and C. Reddy. Trust-Tech-based Global Optimal Methodology for Nonlinear programming. *Recent advances in Global Optimization Methodology*, The Fields Institute Communication series, American Mathematical Society, 2009.

Introduction of TRUST-TECH

- Paper-Theory and Application
  - Hsiao-Dong Chiang, Jaewook Lee. A Dynamical Trajectory-based Hybrid Method for Computing High-quality Optimal Solutions: Method and Theory (A Chapter of the IEEE Press Book). Modern Heuristic Optimization Techniques: Theory and application to Power Systems, IEEE Press Book, 2007.
  - Jaewook Lee, **Hsiao-Dong Chiang**. A Dynamical Trajectory-based Methodology for Systematically Computing Multiple Optimal Solutions of General Nonlinear Programming Problems. *IEEE Transactions on Automatic Control*, 49(6): 888-899, 2004

Introduction of TRUST-TECH

- Paper-Theory and Application
  - **Hsiao-Dong Chiang**, Chia-Chi Chu. A Systematic Search Method for Obtaining Multiple Local Optimal Solutions of Nonlinear Programming Problems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 43(2): 99-109, 1996.
  - Jaewook Lee, **Hsiao-Dong Chiang**. Theory of Stability Regions for a Class of Nonhyperbolic Dynamical Systems and Its Application to Constraint Satisfaction Problem. IEEE Transactions. on Circuits and Systems I: Fundamental Theory and Applications, 49(2): 196-209, 2002.
  - Jaewook Lee, **Hsiao-Dong Chiang**, Singular Fixed-Point Homotopy Method to Locate the Closest Unstable Equilibrium Point for Transient Stability Region Estimate. IEEE Transactions on Circuits Systems II: Express Briefs, 51(4): 185-189, 2004.

Introduction of TRUST-TECH

- Paper-Theory and Application
  - Chandan K. Reddy, **Hsiao-Dong Chiang**. A Stability Boundary based Method for Finding Saddle Points on Potential Energy Surface. *Journal of Computational Biology*, 13(3): 745-766, 2006.
  - Chandan K. Reddy, **Hsiao-Dong Chiang**, Bala Rajaratnam. TRUST-TECH-Based Expectation Maximization for Learning Finite Mixture Models. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 30(7): 1146-1157, 2008.
  - Chandan K. Reddy, Yao-Chung Weng, **Hsiao-Dong Chiang**. Refining Motifs by Improving Information Content Scores using Neighborhood Profile Search. *BMC Algorithms for Molecular Biology*, 1:23, 2006.

TRUST-TECH

- We explain the Trust-Tech framework in solving the following unconstrained nonlinear programming problem.

$$\boxed{\min_{x \in R^n} f(x), \quad f : R^n \rightarrow R, f \in C^2} \tag{1.1}$$

- *f*(*x*) is a nonlinear function with multiple local optimal solutions. All these solutions satisfy:

$$\boxed{\nabla f(x) = 0} \tag{1.2}$$

TRUST-TECH Methodology

– we consider the corresponding dynamical system based on (1.1) :

$$\boxed{\frac{dx}{dt} = -\nabla f(x)} \qquad (1.3)$$

– There is a one-to-one relationship between a stable equilibrium point of (1.3) and an isolated minimum of (1.1) ;

– *xs* is a stable equilibrium point of (1.3) if and only if it is a local optimal solution of the unconstrained optimization problem (1.1).

Theoretical Basis of Trust-Tech Methodology

Prof. Chiang

Introduction of TRUST-TECH

– **Goal:** solving unconstrained nonlinear programming problem(UNLP)
– **Mathematical formulation**:

$$\min c(x)$$
$$x \in R^{1\times n}$$

– **Difficulty:** the multiple local optimal solutions, not easy to find global optimal solution

– **Basic idea & Theoretical foundations (Chiang and Chu):** Consider the following gradient system

$$\dot{x} = -\nabla c(x)$$

Introduction of TRUST-TECH

– **Stable Equilibrium Point and Local Minima (Chiang & Chu 1996)**

$$\textit{Stable equilibrium points} \xleftrightarrow{\text{1-1}} \textit{local minima}$$

– **Development and Characterization of Quasi-stability boundary (Chiang, 1996)**

$$\partial A_p(x_s) \subseteq \bigcup_{\sigma_i \in \partial A_p} \overline{W^s(\sigma_i)} \qquad \sigma_i \textit{ is e.p. on } \partial A_p(x_s)$$

By exploring "**structure of practical stability boundary**", Trust-Tech can locate multiple **local minima** in a deterministic manner

Patents in Optimization Technologies

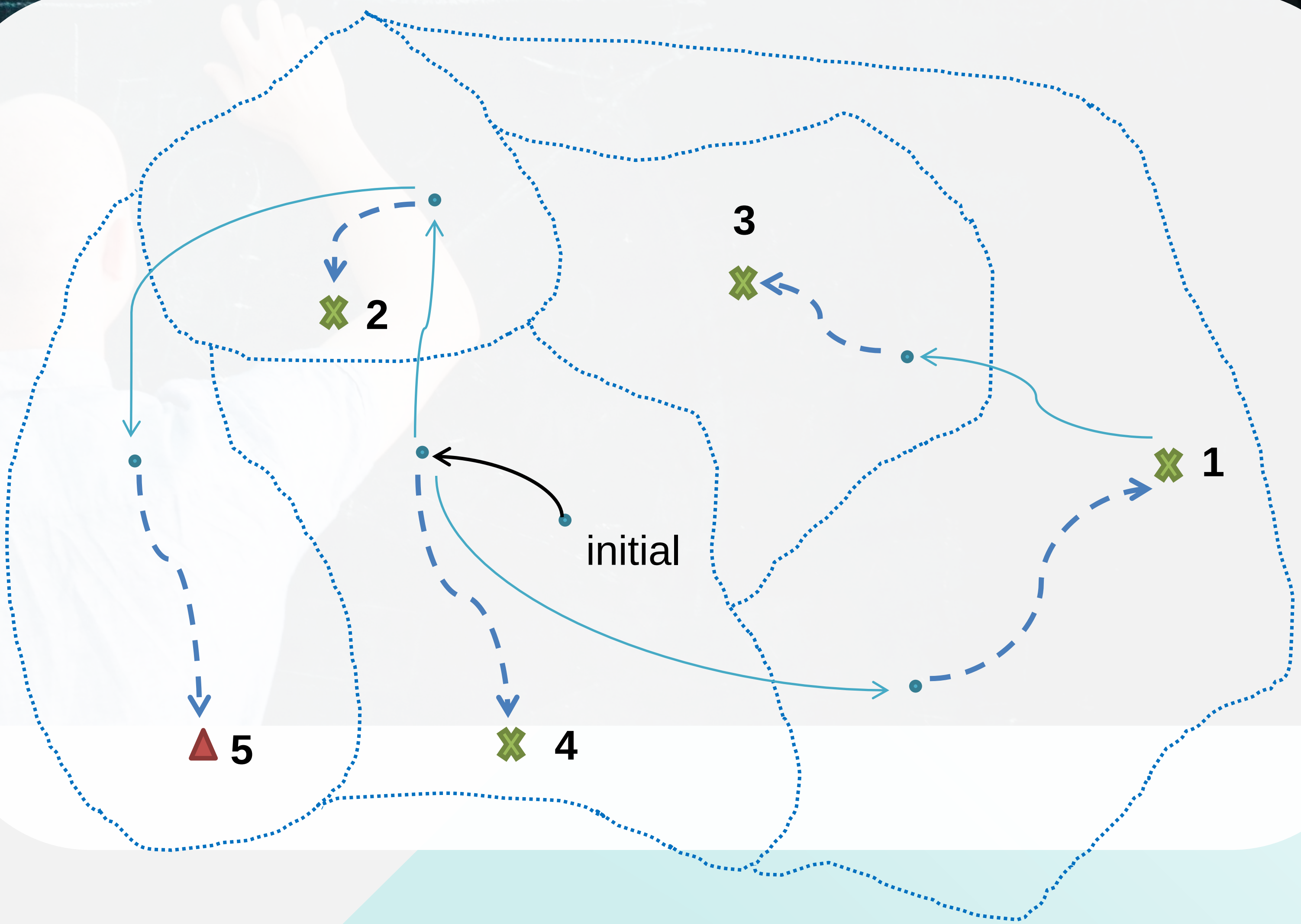
- **U.S. Patent 7,050,953;** “Dynamical Methods for Solving Large-scale Discrete and Continuous Optimization Problems” Date of Patent, May 23, 2006 (Inventors: Hsiao-Dong Chiang, Hua Li)
- **U.S. Patent 7,277,832;** “Dynamical Method for Obtaining Global Optimal Solution of General Nonlinear Programming Problems”, Date of Patent, Oct. 2, 2007, (Inventor: Hsiao-Dong Chiang)

Patents Pending (2)

-

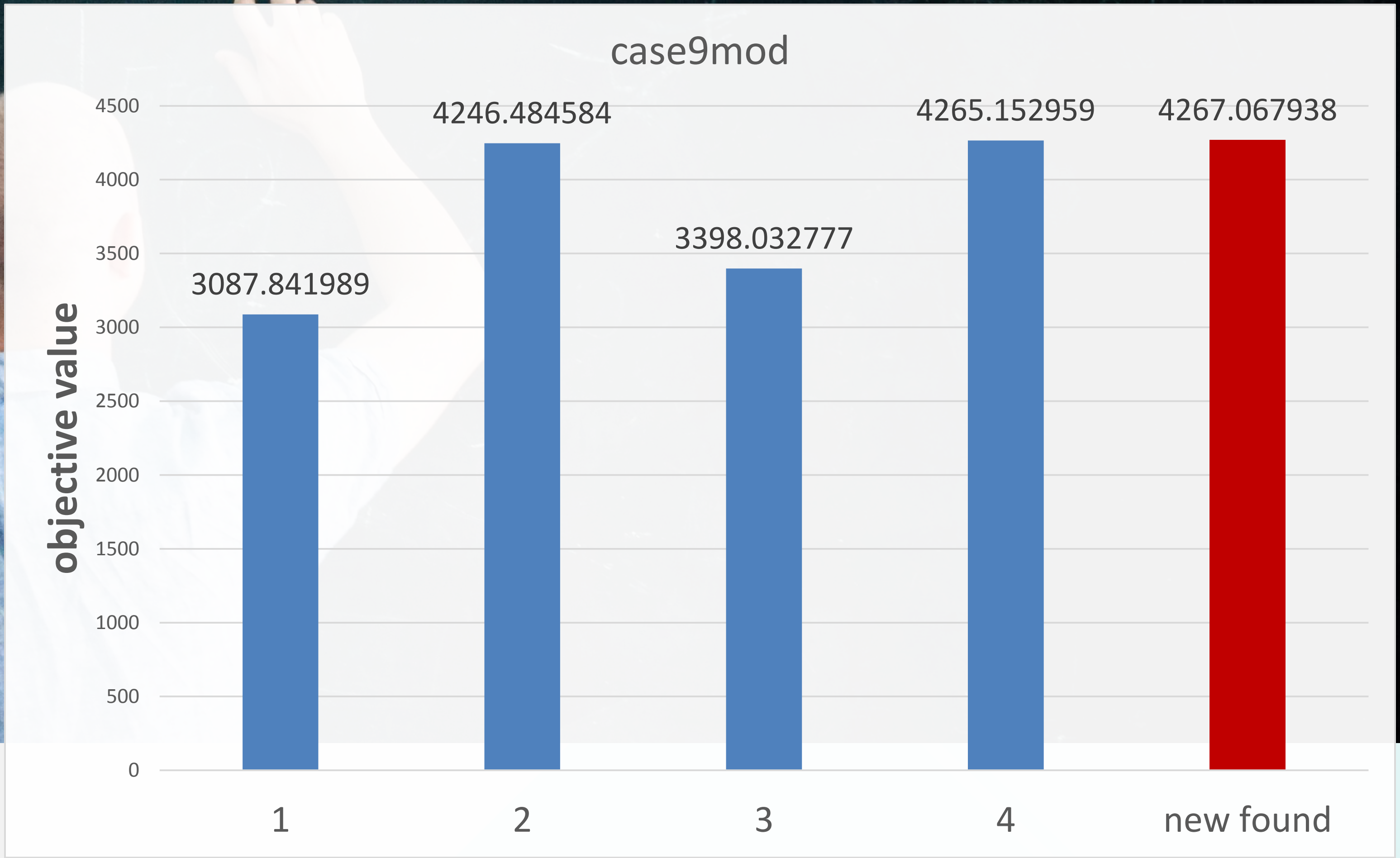
- **U.S. Patent Application (2013),** "PSO-assisted Trust Tech Methodology for Nonlinear Optimization", Dr. Hsiao-Dong Chiang (Ithaca, NY, USA)

How to find multiple solutions



Case 9-bus

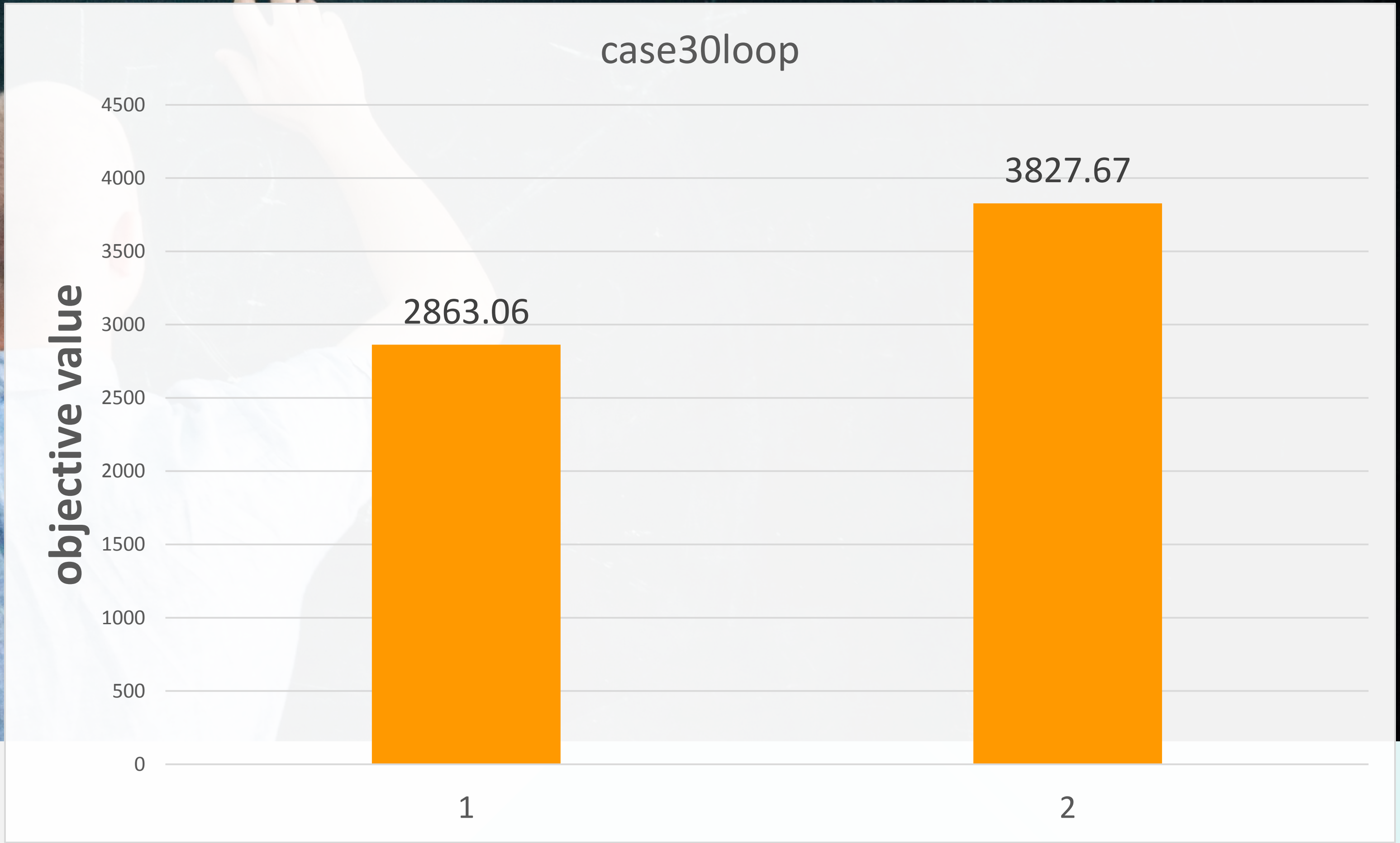
OPF Multiple Solutions Examples



Case 9-bus

(5 local solutions: the difference can be 40%)

OPF Multiple Solutions Examples



Case 30-loop

(30-bus test system, 2 local optimal solutions and the difference can be 33%)

CONCLUSION

SuperOPF

is an

advanced & **comprehensive**

ACOPF solver

needed by modern power systems.

CONCLUSION

Global-OPF

= SuperOPF + Trust-Tech